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Tracer Relations in Variable Flow

by

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where $\Phi(t)$ and $\Phi_{in}(\tau)$ are the output and input fluxes, respectively; g_F is the weighting function for the mass flux, i.e. it represents the transit time distribution of the mass flux which results from and instantaneous injection of tracer at the time τ; λ is the decay constant of a tracer or a pollutant; and t and τ are the times of leaving and entry, respectively. After some substitutions and rearrangements the final formula reads

$$c(t) = \frac{1}{V(t)} \int_{-\infty}^{t} c_{in}(\tau) Q_{in}(\tau) C_I(z) \exp\,[-\lambda(t-\tau)]\, d\tau \tag{2}$$

where $c(t)$ and $c_{in}(\tau)$ are the output and input concentrations, $V(t)$ is the total volume of mobile water in the system, Q_{in} is the volumetric inflow rate, and $C_I(z)$ is the dimensionless concentration resulting from an instantaneous injection, expressed in dimensionless time. This function represents the assumed, or fitted, type of the model. For the two most common models, i.e. good mixing and dispersive, it reads

$$C_I(z) = \exp(-z), \text{ and} \tag{3}$$

$$C_I(z) = [Pe/(4\pi z^3)]^{1/2} \exp\,[-Pe(1-z)^2/(4z)] \tag{4}$$

where Pe is the Peclet number (a fitting parameter) and z is

$$z = \int_{-\infty}^{t} \frac{Q(t')}{V(t')}\, dt' \tag{5}$$

For an instantaneous injection in artificial tracing eq. 2 simplifies to

$$c(t) = MC_I(z)/V(t) \tag{6}$$

where M is the injected mass or activity, and z is integrated from zero, which means that the time scale starts at the injection time.

Eq. 2 is applicable to environmental tracers and pollutants appearing uniformly in the total inflow, Q_{in}. In other cases, instead of the $C_{in}\,Q_{in}$ term, one should use directly an estimate of the input flux, Φ_{in}.

It is clear from eq.2 that a long record of Q_{in} is needed whereas this function is unknown in principle. However, in groundwater systems the turnover time can be assumed to be constant. Then

$$Q_{in}(t) = t_d \frac{dQ(t)}{dt} + Q(t) \tag{7}$$

where t_d is the dynamic turnover time of the variable part of the system. The total volume of the system is

$$V(t) = V_d(t) + V_m \tag{8}$$

where V_m is the minimum volume which becomes stagnant when $V_d(t) = 0$. For small systems, the dynamic turnover time, t_d, is easily determined from the analysis of $Q(t)$ in periods of $Q_{in}(t) = 0$. Knowing t_d one can easily determine the $V_d(t)$ function from

$$t_d = V_d(t)/Q(t) \tag{9}$$

For the exponential model (eq.3), the minimum volume, V_m, is the only fitting parameter, whereas for the dispersive model (eq.4), the Peclet number is additionally fitted.

In the case of a long-term movement of a tracer through a fissured aquifer with a porous matrix, the tracer moves as if the flow took place in the whole volume of water [8], i.e. both in the fissures (mobile water volume) and in the microporous matrix (stagnant water volume). For such an aquifer, instead of V(t), the following expression should be put into the previous equations

$$V'(t) = RV(t) \tag{10}$$

where R is the ratio of the total porosity to the fissure porosity.

Experimental

Area of investigations and basic hydrologic data.

Data from a small retention basin, Lange Bramke, Upper Harz Mts., Federal Republic of Germany, are considered. The area of the basin is 0.76 km^2, 90% of which is forested. The groundwater reservoir consists of three components [3,4]: (1) the unsaturated zone of residual weathering and allochtonic Pleistocene solifluidal materials; (2) fractured Lower Devonian sandstones, quartzites, and slates; and (3) gravels, pebbles, and boulders located in the valley bottom.

The rainfall on and surface outflow from the basin have been measured since 1964. The following average figures can be given:

Precipitation 1300 mm/a
Surface outflow 700 mm/a = 17.1 l/s = 0.532 x $10^6 m^3/a$.

The total infiltration is estimated at 750 mm/a, which means that 50 mm/a is supposed to discharge underground to the adjacent basin. The direct run-off has been found to be abt. 11.5% of the surface outflow [3]. Thus, the groundwater run-off in the surface outflow is 620 mm/a = 15.1 l/s = 0.471 x $10^6 m^3/a$.

20-year observations showed that the system has several components of the dynamic turnover time [6]. The weighted mean value is 8.7 days. The monthly values of the precipitation, surface outflow, and inflow rates for the period of five years are shown in Fig. 1. The inflow rates were calculated from eq.7 using the above-given mean t_d-value. A similar t_d-value can be found directly from eq.7 by trial-and-error procedure, as the value which gives no negative $Q_{in}(t)$. The variable flow curves were constructed starting from 1964, when the discharge observations were initiated. For the earlier period the average value was used.

Tritium data and models.

Tritium samples were taken at the discharge station at times when there was no surface run-off. Thus, they represent the groundwater run-off component in surface outflow. This also means that the part of the system which discharges underground from the basin is excluded from further considerations. A five-year record of tritium data in the outflow exists as shown in Fig.2. For comparison, the tritium data in the input are also shown. Theoretical concentrations curves of the best fit are given for the assumed exponential model, both for the steady state approximation and for variable flow.

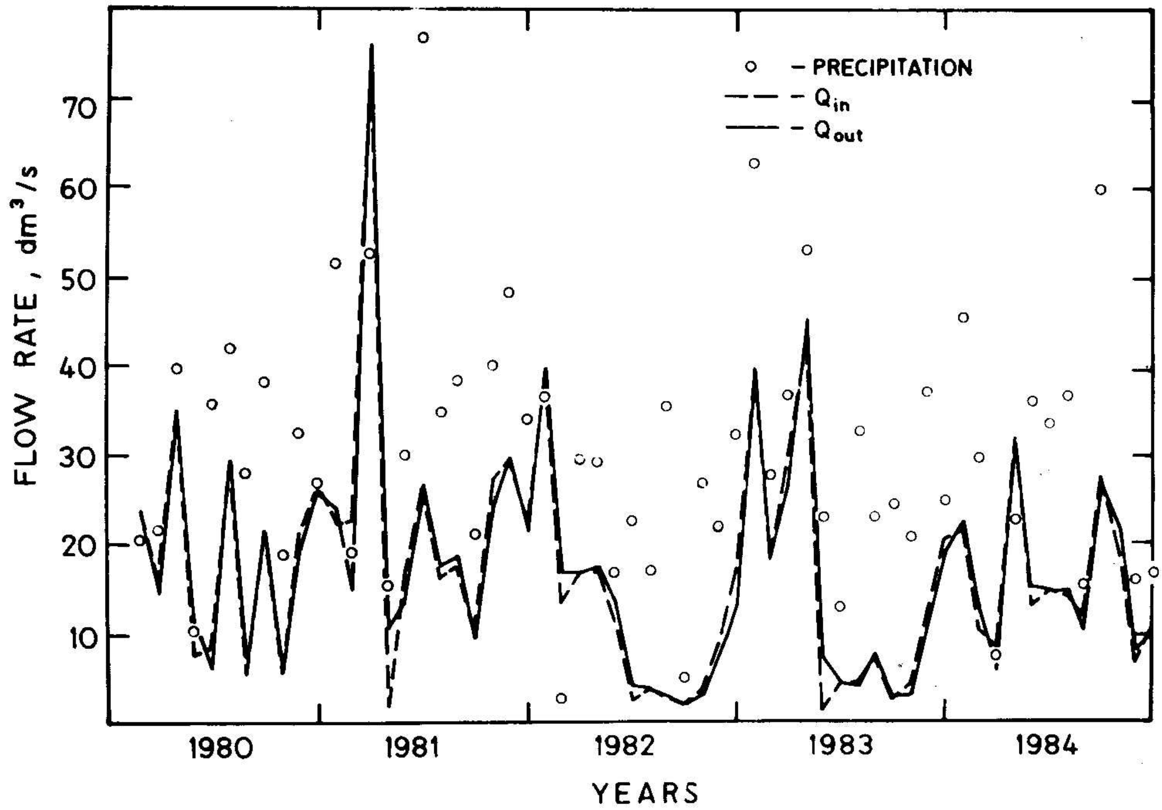

Fig. 1: Measured outflow rates from the Lange Bramke basin and calculated (eq.7) inflow rates for the period of 1980-84. The precipitation rates given for comparison. Tritium data in the outflow from the Lange Bramke basin and fitted theoretical curves. Tritium in the precipitation given for comparison.

The parameters of the fitted models are presented in Tab. 1. In the steady state approach, the a-parameter represents the ratio of the summer to winter infiltration coefficients [2,7] whereas the t_o-parameter represents the mean turnover time [7]. The accuracy of fitting is represented by

$$\delta = [\sum_{i=1}^{n} (cm_i - c_i)^2]^{1/2}/n \tag{11}$$

where cm_i and c_i are the measured and calculated tritium concentrations, respectively, and n is the number of measurements. The δ values given in Table I show that the variable flow approach gives slightly better fitting, but the difference is not essential. As both approaches give close accuracy of fitting it is not astonishing that the final values of parameters obtained from them are also close, in spite of large variations of the flow rates (see Fig.1). The total volume of the system is $0.90 \times 10^6 m^3$ which divided by the area of the basin gives the average height of water equal to 1.18m, whereas the mean volume of the variable part is $Qt_d = 1.28 \times 10^4 m^3$ which gives the average height of variable water reservoir equal to 0.017m.

As mentioned earlier, a part of the system consists of the fissured rock, thus the mobile water volume is undoubtedly lower than the obtained total volume. The correction coefficient remains unknown so far, because both the micropore porosity and the fissure porosity are not known.

Tab. 1: Known (or assumed), fitted, and calculated parameters of the exponential model of the Lange Bramke basin, both for the steady and variable flow approaches

Flow	Parameters			Accuracy of fitting
	Known	Fitted	Calculated	δ, T.U.
	a	t_o, years	$\overline{V}=\overline{Q}t_o$, $10^6 m^3$	
Steady	0.5	2.2	1.04	0.99
	1.0	1.8	0.85	1.00
	t_d, days	V_m, $10^6 m^3$	$\overline{V}=V_m+Qtd_o$, $10^6 m^3$	
Variable	8.7	0.89	0.90	0.85

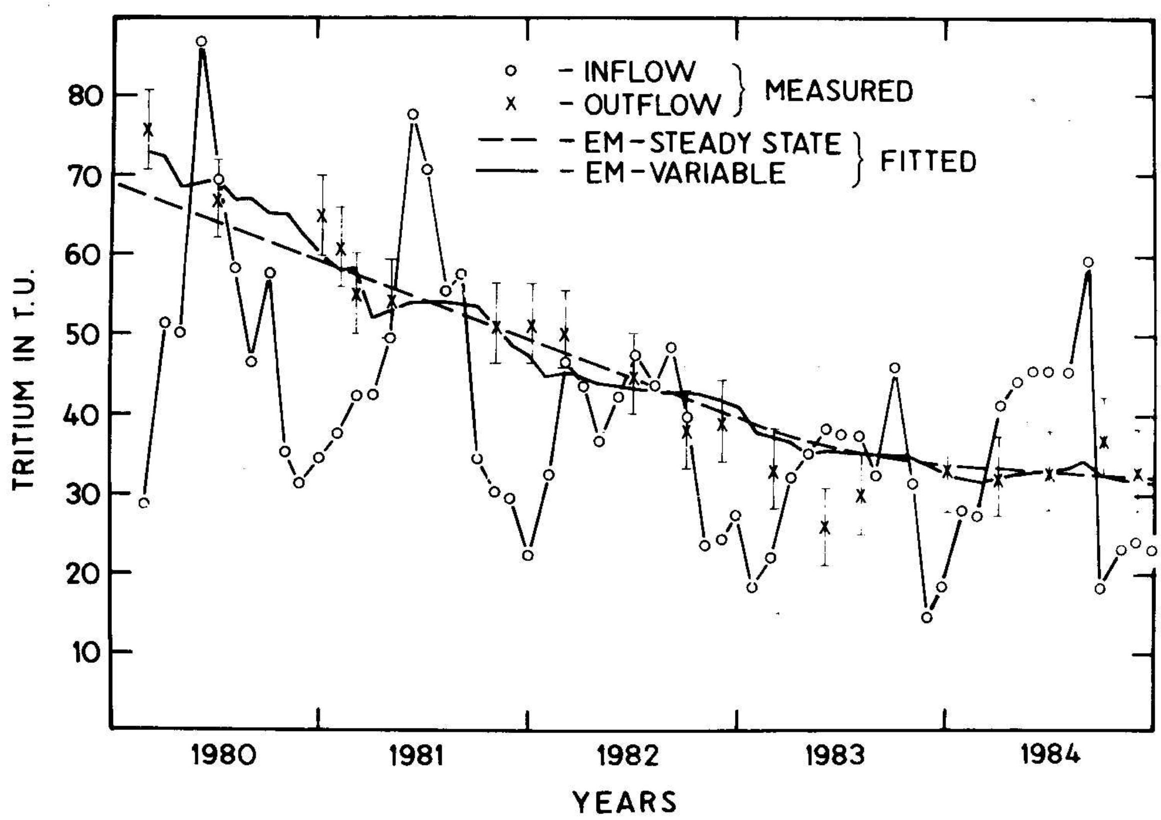

Fig. 2: Tritium data in the outflow from the Lange Bramke basin and fitted theoretical curves. Tritium in the precipitation given for comparison.

Conclusions

It has been shown that the proposed approach is applicable to groundwater systems with variable flow and volume. However, in the presented example, in spite of high variations of the flow rates, this approach gave practically the same values of the final parameters as the steady-state approximation. This astonishing agreement of two different approaches can easily be understood if one notes that the variable volume of the investigated system is a small fraction of the total volume. Thus, it may be suspected that in all similar cases the steady state approximation is applicable, in spite of possible high changes in flow rates.

This work also proves that earlier models of the Lange Bramke basin [4,8] are essentially correct. In these models, the steady state approximation was applied to the separated parts of the basin. A somewhat lower total volume obtained within this work, in comparison with the previous estimates, results simply from a better fit.

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